'Settling time' and 'memory': two concepts to simplify the dynamic analysis of the flat-plate collector

M. F. El-Refaie and M. A. Hashish*

Concepts of 'settling time' and 'memory' in a solar collector are introduced and explained; they are shown to characterize the collector and the fluid-flow collectively. These two characteristics are then used to develop a simplified method for the calculation of the fluid temperature within a flat-plate collector. Under all conditions, the proposed method can be used with any arbitrary degree of accuracy. The temperature is given, in a normalized form, as a function of both time and position along the collector. The numerous design and operating variables are lumped in a limited number of dimensionless groups

Keywords: *solar collectors, thermal design, mathematical modelling*

Realistic analyses of the flat-plate solar collector, as a dynamic system operating under unsteady conditions, have been presented recently. Closed-form mathematical expressions have been derived to give the fluid and absorber-plate temperatures as functions of both time and position along the collector[1]. These expressions, however, are quite complicated and their evaluation requires considerable effort and time[2]. It was such complexity that paved the way for the approximate steady-state models which have been adopted for many years[3-6].

This paper introduces two new concepts, the 'Zero-Input Settling Time' and the 'Memory', which are used as a basis to develop a simplified method for evaluating the fluid temperature at any position along the collector and at any instant of time during the sunlight hours or after sunset. The results are compared with those obtained by other exact elaborate techniques, and are found to be accurate under all conditions.

The most simple collector design, with the fluid-stream bathing the entire rear surface of a flat absorber plate[4], is considered in this study. Nevertheless, the technique presented and the results obtained can be applied to any form of uniformly-irradiated collector with only minor changes in the formulation of the problem.

The operation of the collector is assumed here to start with sunrise. The method, however, can be modified simply for any other starting time. This may be the case in practice if operation is confined to a part of sunlight hours during which the output is in excess of a certain predetermined minimum[5]. The study is also extended to include the cases if the collector operation is continued for a certain period after sunset. This may theoretically be feasible if the collector absorber has a very high thermal inertia and/or the fluid-stream heat capacity is low[1]; otherwise nocturnal operation will yield a zero, or even negative output[2]. The after-sunset operation, which is impractical for a simple collector, is presented to complete the analysis; moreover, it may be important for the study of collectors with integrated storage.

Dynamic fluid-temperature profile

The simple flat-plate collector is shown in Fig 1. The fluid temperature at any position x along the collector and at any time t has been obtained in the following form[1,2]:

$$T_f(X, t) = T_i\, e^{-\sigma X}[1 - F(\alpha t, \beta X)] + \gamma \int_{z_1}^{z_2} e^{-\gamma z} F(\psi z, \phi X) R(t - z)\, dz \qquad (1)$$

The detailed derivation of this formula, and the assumptions upon which it is based, are given elsewhere[1]. The various quantities in Eq (1) are defined as:

1. $X = x/L \qquad 0 \leq X \leq 1 \qquad (2)$
   is the normalized distance along the collector.

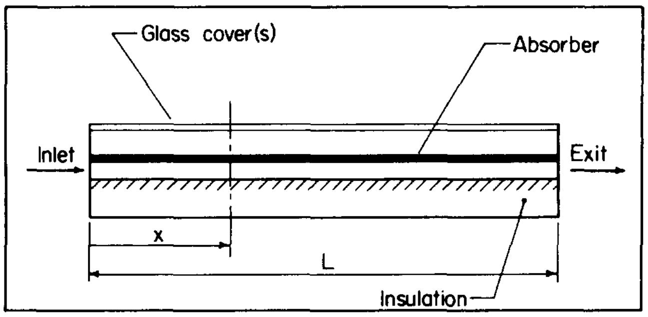

Fig 1 A flat-plate solar collector

* Faculty of Engineering, Cairo University, Giza, Egypt
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2. t is the time reckoned from sunrise, h
3. $\gamma = U/m_p c_p \quad h^{-1}$ (3)
4. $\phi = h/\dot{m}_f c_f$ dimensionless (4)
5. $\psi = h/m_p c_p = \gamma h/U \quad h^{-1}$ (5)
6. $\alpha = \psi + \gamma \quad h^{-1}$ (6)
7. $\sigma = \phi\gamma/\alpha$ dimensionless (7)
8. $\beta = \phi\psi/\alpha = \phi/(1+\gamma/\psi)$ dimensionless (8)
9. $T_f = (\theta_f - \theta_a)/(\theta_e - \theta_a) \leq 1$ dimensionless (9)

   where $\theta_f(X, t)$ is the actual fluid temperature (°C), θ_a is the average ambient air temperature (°C), and T_f is a normalized fluid temperature. The equilibrium or stagnation temperature θ_e is the theoretical maximum temperature attained under no-flow condition and continuous exposure to the maximum radiant input, thus:

$$\theta_e = \theta_a + H_{max}/U \tag{10}$$

   and

$$T_f = (\theta_f - \theta_a)U/H_{max} \tag{11}$$

   T_i is the normalized fluid temperature at inlet to collector.

10. The function $F(a, b)$ is defined by:

$$F(a, b) = e^{-a}\int_0^b e^{-y} I_0(2(ay)^{1/2})\, dy \tag{12}$$

   where I_0 is the zero order modified Bessel function of the first kind.

11. $R(t) = H(t)/H_{max} \quad 0 \leq R \leq 1$ (13)

   is the normalized radiant input (Fig 2).

12. The limits of the convolution integral z_1 and z_2 are given by:

$$z_2 = t \tag{14}$$

$$z_1 = \begin{cases} 0 & \text{for } t \leq t_{ss} \\ t - t_{ss} & \text{for } t \geq t_{ss} \end{cases} \tag{15}$$

Notation

a	First argument of the function $F(a, b)$, dimensionless
a_i	Coefficients of the radiant input polynomial, Eq (31), $i = 0, 1, \ldots, m$
A_i	Coefficients of the polynomial $R(t-z)$, given by Eq (33), $i = 0, 1, \ldots, m$
b	Second argument of the function $F(a, b)$, dimensionless
$\bar{b}$	Switching value of b, given by Eq (22), dimensionless
B_i	Coefficients defined by Eq (36), $i = 0, 1, \ldots, m+n$
C	The term independent of 'a' in the expansion of $F(a, b)$ given by Eq (24), dimensionless
C_i	Coefficients defined by Eq (25), $i = 0, 1, \ldots, n$
c_f	Specific heat of the fluid, Joule/kg °C
c_p	Specific heat of the absorber-plate material, Joule/kg °C
D_i	Coefficients defined by Eq (35), $i = 0, 1, \ldots, m$
F	A function of two variables, defined by Eq (12), dimensionless
G_i	Coefficients defined by Eq (39), $i = 0, 1, \ldots, m+n$
h	Plate-to-fluid heat transfer coefficient, W/m^2 °C
H	Instantaneous rate of radiant energy absorbed by the absorber plate, W/m^2
H_{max}	Maximum value of H, W/m^2
I_0	Modified zero-order Bessel function of the first kind
L	Length of collector (Fig. 1), m
m	Degree of the polynomial expressing the normalized radiant input, Eq (31)
m_f	Fluid mass rate of flow per unit of plate area, kg/m^2 h
m_p	Mass of the absorber plate per unit area, kg/m^2
M	Memory at any section of the collector, defined by Eq (19), h
n	Degree of the polynomial in Eq (24)
P_i	Coefficients defined by Eq (40), $i = 0, 1, \ldots, m$
R	Normalized radiant input, defined by Eq (13) and shown in Fig 2, dimensionless
t	Time reckoned from sunrise, h
t_{set}	Settling time at any section of the collector, defined by Eq (18), h
t_{ss}	Sunset time, h
T_f	Normalized fluid temperature, defined by Eq (9) or (11), dimensionless
T_{f1}	Zero-input temperature response, dimensionless
T_{f2}	Zero-state temperature response, dimensionless
T_i	Normalized fluid temperature at inlet to the collector, dimensionless
U	Overall collector heat-loss coefficient, W/m^2 °C
w_b	Window, shown in Fig 5, dimensionless
x	Distance along the collector (Fig 1), m
X	Normalized distance along the collector, defined by Eq (2), dimensionless
z_1	Lower limit of the convolution integral in Eq (1), h
z_2	Higher limit of the convolution integral in Eq (1), h
α	Parameter defined by Eq (6), h^{-1}
β	Dimensionless group defined by Eq (8)
γ	Parameter defined by Eq (3), h^{-1}
ε	Value of $F(a, b)$ at $a = w_b$, dimensionless
θ_a	Ambient air temperature, °C
θ_e	Equilibrium or stagnation temperature, defined by Eq (10), °C
θ_f	Fluid temperature at any time and any position along the collector, °C
σ	A dimensionless group defined by Eq (7)
ϕ	A dimensionless group defined by Eq (4)
ψ	A parameter defined by Eq (5), h^{-1}

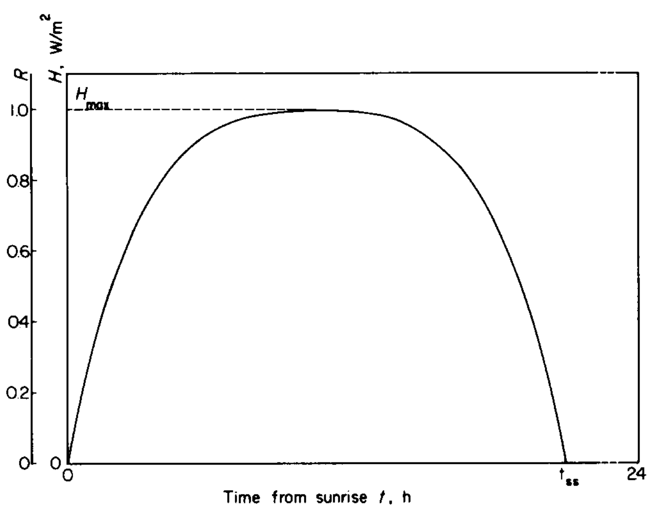

Fig 2 Daily variation of the radiant energy input

Interpretation of the dynamic fluid-temperature response

The fluid temperature $T_f(X, t)$ may be considered as the superposition of two components, represented by the two terms on the right-hand side of Eq (1). The first component is due to the inlet fluid temperature T_i, and will be called the inlet-temperature component or the zero-input response. The second component is induced by the radiant input, and will be known as the radiation component or the zero-state response. The two components will be denoted by $T_{f1}(X, t)$ and $T_{f2}(X, t)$ respectively.

Eq (1) implies that the zero-input response is always dependent on time no matter how long has elapsed after sunrise. Considering the zero-state response, it can be seen from Eqs (1) and (14) or (15) that the convolution integral spans a part of the daily radiation profile extending from the particular moment of interest backward to sunrise (Fig 3). This means that all the history of radiant input, up to a particular instant, contributes to the instantaneous value of fluid temperature. In other words, the collector keeps a record of, or compiles, the radiant-input variation since sunrise.

Based on Eq (1), different computational methods have been proposed for calculation of the fluid temperature[2]. All the suggested methods, however, are inevitably lengthy. At any section of the collector, the required computational effort increases as the time t increases. This called for the development of a simplified temperature expression which would reduce the computational work. This is achieved, in this work, by introducing two new concepts, settling time and memory.

Settling time and memory

Study of $F(a, b)$, defined by Eq (12) and displayed graphically in Fig 4, is crucial to developing an approximation for the dynamic response expressed by Eq (1). This monotonic decreasing function will be considered only within the finite interval $0 \leq a \leq w_b$, where the function is greater than ε and ε is a given arbitrary accuracy. This is shown in Fig 5. The sub-

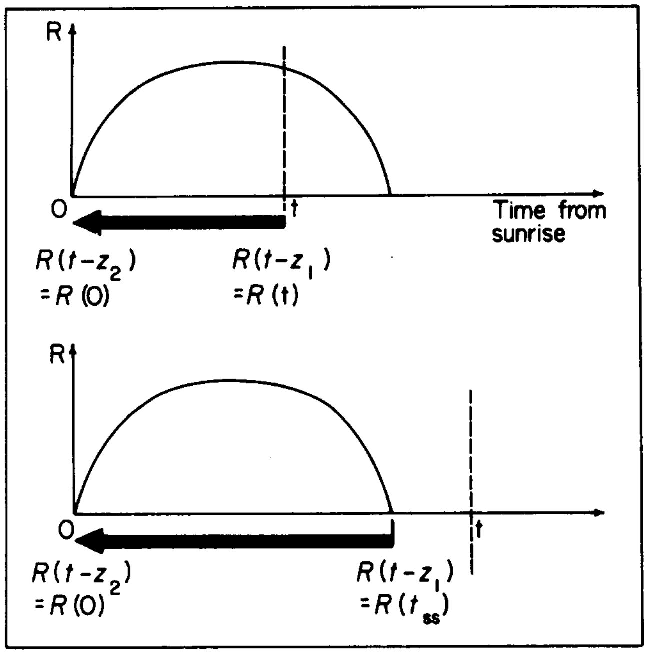

Fig 3 The part of the daily radiation profile spanned by the convolution integral in Eq (1) (*or contributing to the fluid temperature at time* t)

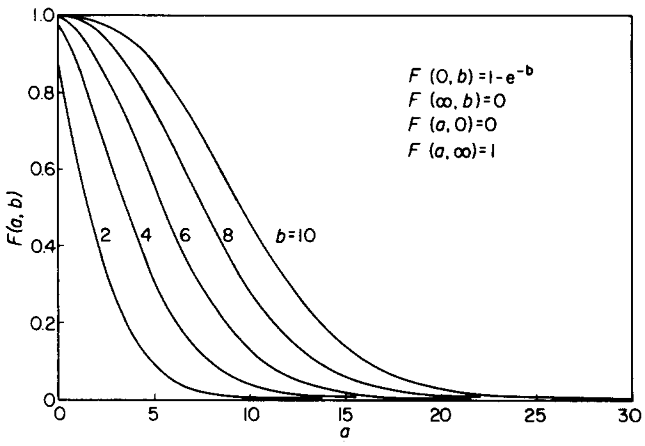

Fig 4 Variation of F(a, b) *with* a *for various values of* b

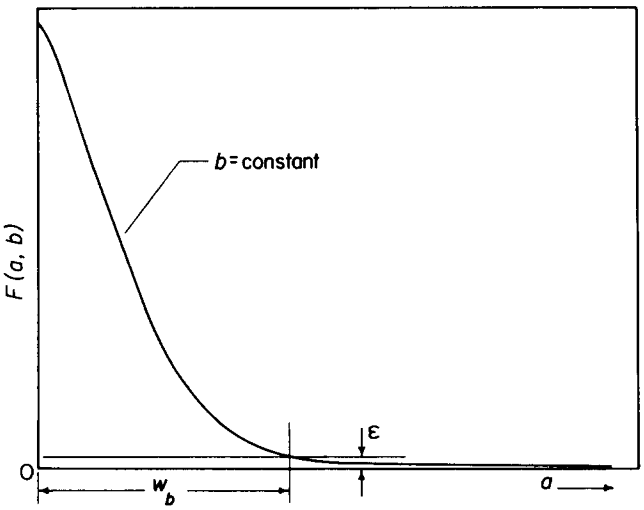

Fig 5 Window of F(a, b)

script of w stands for the second argument of the function $F(a, b)$. This is the same as if the graph of $F(a, b)$ is being observed through a rectangular-shaped window having a width w_b and its left edge coincident with the ordinate $a = 0$. Outside this window the value of $F(a, b)$ is considered to be zero, ie:

$$F(a, b) \simeq 0 \qquad \text{for } a > w_b \tag{16}$$

The width of the window w_b increases as the second argument b increases and/or the error ε decreases.

Based on the window concept, both components of the fluid temperature can be simplified.

Inlet temperature component (zero-input response)

After a certain time t from sunrise and for a given accuracy ε, the product (αt) reaches the value of the window $w_{(\beta X)}$. For $\alpha t > w_{(\beta X)}$, the function $F(\alpha t, \beta X)$ can practically be considered to be zero. Accordingly, the zero-input response reduces to:

$$T_{f1} = T_i \, e^{-\sigma X} \qquad \text{for } \alpha t > w_{(\beta X)} \tag{17}$$

Physically, this means that, at any position along the collector, the zero-input response reaches a limiting or stabilized value after a certain period of time which will be called 'The zero-input settling time' or simply 'The settling time'; and is given by:

$$t_{set} = w_{(\beta X)}/\alpha \tag{18}$$

The window $w_{(\beta X)}$ becomes wider and consequently the settling time becomes larger as the distance X increases. On the other hand, t_{set} is independent of the initial temperature T_i. A graphical explanation of the settling time is shown in Fig 6.

Radiation component (zero-state response)

The domain of the function $F(\psi z, \phi X)$ will be confined to a window $w_{(\phi X)}$, i.e. the function $F(\psi z, \phi X)$ will only be considered within the time interval $0 \leq z \leq M$, where:

$$M = w_{(\phi X)}/\psi \tag{19}$$

For any value of time t (Fig 7), the product $[F(\psi z, \phi X) \cdot R(t - z)]$ will have non-zero values over a certain interval of the dummy argument z. Outside this interval, the integrand of the convolution integral is equal to zero either because $F = 0$ or $R(t - z) = 0$. Accordingly, the upper limit of integration z_2 should be readjusted such that the convolution integral will span only the required interval of z. The new integration limits are shown opposite to each case in Fig 7.

According to the limits of integration given in Fig 7, the part of the actual daily radiation profile spanned by the convolution integral is shown in Fig 8 for the corresponding cases displayed in Fig 7. Therefore, the effective part of the radiation profile is confined to a time band having a width M and continuously sliding such that its right edge coincides with the time under consideration.

Fig 8 clarifies the physical significance of the time interval M. At any section of the collector, the instantaneous value of the zero-state response at any instant of time is affected by the radiant energy input during an interval of time M prior to this particular

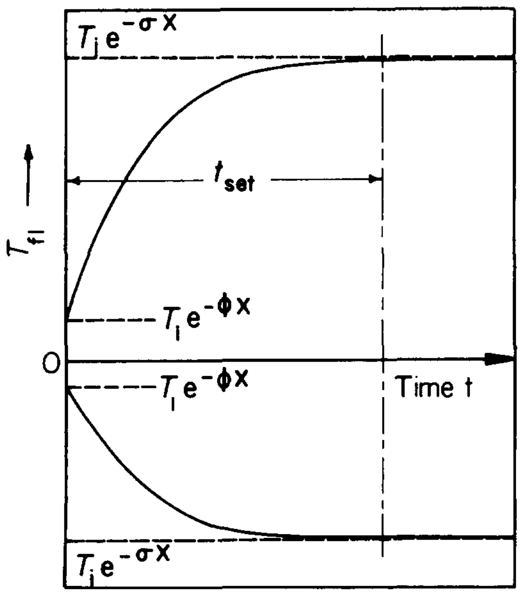

Fig 6 Graphical explanation of settling time at one collector-section. Two different inlet temperatures are considered and all other parameters are the same

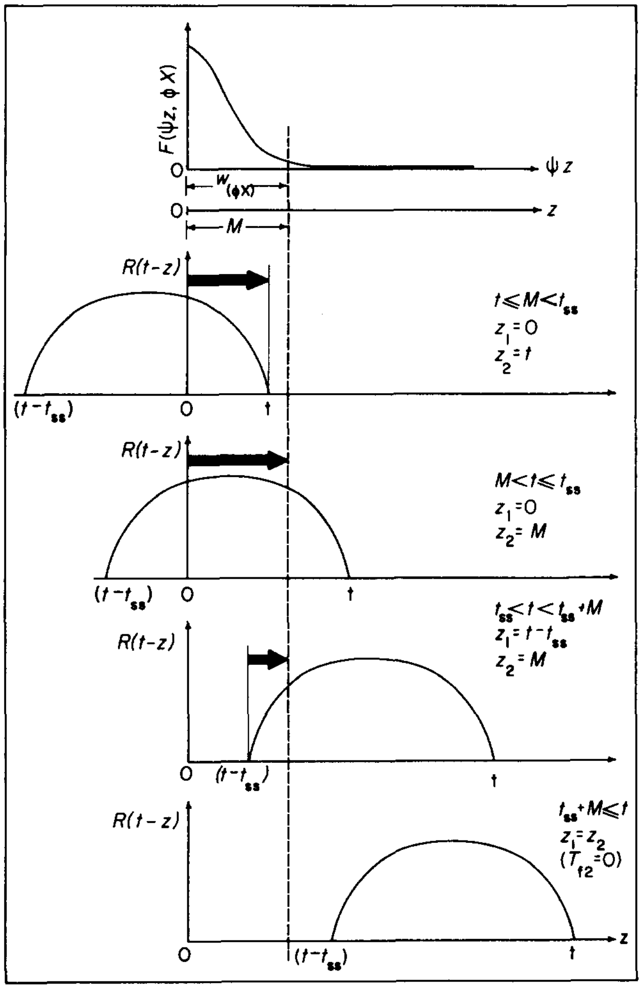

Fig 7 Effective range of the convolution integral in Eq (1)

instant. The radiant energy input at earlier times has almost no contribution to the instantaneous fluid temperature at that particular instant. This implies that the system has a limited memory which goes back for an interval of time equal to M only. All radiation history which took place before this time is almost forgotten. Accordingly, M is known as the 'Memory'. To be more concise, M is the memory at one particular section of the collector. The memory M increases as we move away from collector inlet, ie as X increases; this can be easily seen from Eq (19) and Fig 4. Note that memory is independent of the shape of the daily radiation profile.

Relative magnitudes of settling time and memory

Eq (8) shows that the dimensionless parameter β is always less than ϕ. Thus for any permissible error ε and at any section of the collector, the window $w_{(\phi X)}$ is always greater than the window $w_{(\beta X)}$. At the same time, Eq (6) shows that ψ is always less than α. Accordingly, and from Eqs (18) and (19), $M > t_{set}$, ie the memory at any section is always greater than the settling time.

When the fluid temperature is to be calculated at a certain distance X and time t, one or both of the temperature components on the right-hand side of Eq (1) may be simplified as explained before. This depends on the value of the time t relative to t_{set} and M. In this respect, we may have any of the following three possibilities:

Time, t	Simplifiable component(s)
$t < t_{set} < M$	—
$t_{set} \leq t < M$	T_{f1}
$t_{set} < M \leq t$	T_{f1}, T_{f2}

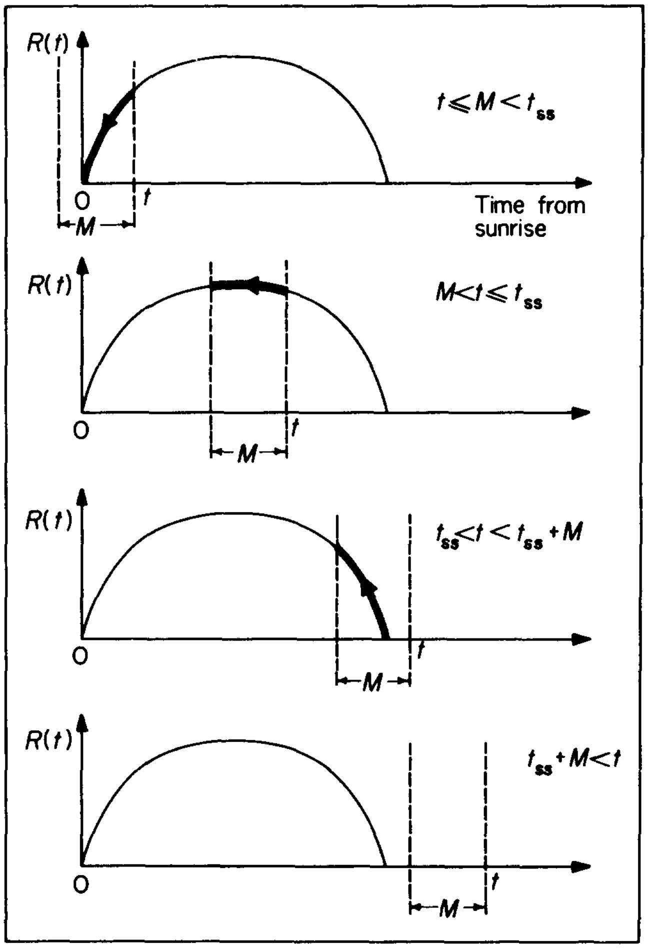

Fig 8 The part of the daily radiation profile effectively contributing to the fluid temperature at time t

Determination of settling time and memory

A computer program has been written to evaluate the window w_b, from Eq (12), for different values of b and ε; the results are given in Fig 9. The window w_b is equal to zero if the maximum value of $F(a, b)$, ie $1-e^{-b}$, is less than ε. This corresponds to values of $b \leq -\ln(1-\varepsilon)$.

For the sake of computational simplicity, the window $w_b(b, \varepsilon)$ is approximated by the two straight lines shown dotted in Fig 9. The equations representing these two straight lines are:

$$w_b = -24(\log \varepsilon + 1)b \qquad b \leq \bar{b} \qquad (20)$$

and:

$$w_b = 2.2\dot{2}b - 4(\log \varepsilon + 1) \qquad b \geq \bar{b} \qquad (21)$$

The switching point from Eq (20) to Eq (21) is at $\bar{b}$ and is given by:

$$\bar{b} = [6 + 0.5\dot{5}/(\log \varepsilon + 1)]^{-1} \qquad (22)$$

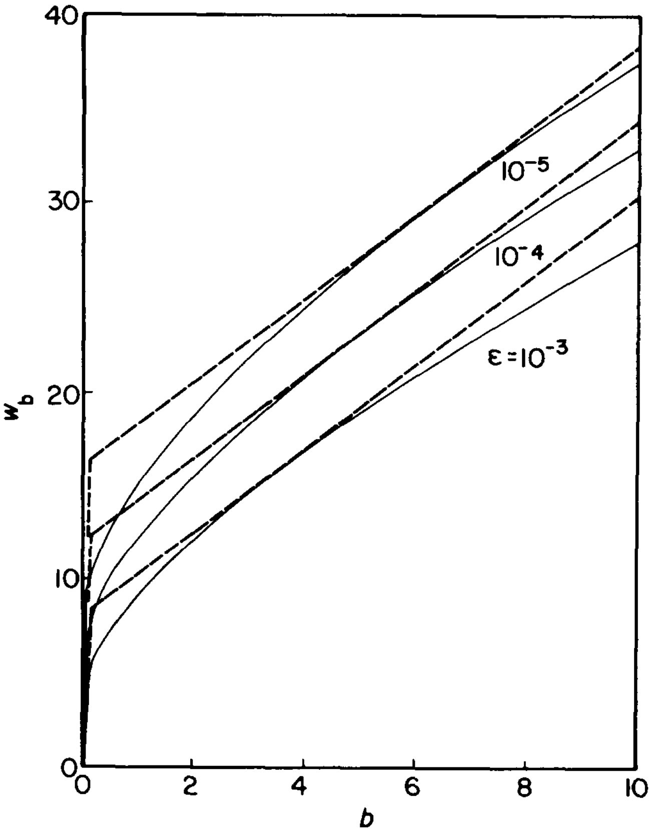

Fig 9 Variation of the window w_b *with* b *for different values of* ε

At any section of the collector, the settling time and memory can be determined by calculating the windows $w_{(\beta X)}$ and $w_{(\phi X)}$ from Eq (20) or (21) and then dividing them by α and ψ respectively.

Note that, for fixed values of b and ε, the window w_b obtained from Eq (20) or (21) is greater than the actual value shown by the continuous curve in Fig 9. Accordingly, the two straight lines approximation implies a reduction of the error below the specified value ε.

Factors affecting the settling time and memory

It can be deduced from Eqs (4)–(6), (8), (18), and (19) and Fig 9 that both the settling time and memory, at any section, increase with the increase of the ratio $m_p c_p / \dot{m}_f c_f$. Moreover, the memory increases as the plate-to-fluid heat transfer coefficient h decreases; and the settling time increases as the overall heat-loss coefficient U decreases. The difference between the settling time and memory decreases with the decrease of U; they tend to be equal in the theoretical case of zero loss.

Considering typical actual values for the different parameters, the practical range of the memory and settling time can be estimated. The longest values are expected to be in the order of one hour; such a value will be met at the exit of a collector with high absorber heat capacity, heavy insulation, and low fluid-stream heat capacity. On the other hand, they may be as short as one or two minutes close to the inlet of a collector with low absorber heat capacity, light insulation, and high fluid-stream heat capacity.

Approximation of F

A further simplification of the fluid-temperature calculation can be achieved through approximating the function F by a relatively simple expression. It can be shown from Eq (12), by expanding the Bessel function and integrating, that:

$$F(a, b) = 1 - e^{-b} - e^{-b} \sum_{i=1}^{\infty} \frac{b^i}{i!} \left[1 - e^{-a} \sum_{j=0}^{i-1} \frac{a^j}{j!} \right] \qquad (23)$$

The infinite summation may be truncated to $(n+1)$ terms; consequently, Eq (23) can be rearranged into an approximate form given by:

$$F(a, b) \simeq C + e^{-a} \sum_{j=0}^{n} C_j a^j \qquad (24)$$

where:

$$C_j = e^{-b} \sum_{k=j}^{n} \frac{b^{k+1}}{(k+1)!\,j!} \qquad j = 0, 1, \ldots, n \qquad (25)$$

and:

$$C = 1 - (e^{-b} + C_0) \qquad (26)$$

The coefficients C_j of the polynomial are interrelated by the recurrence relation:

$$C_j = (j+1)C_{j+1} + e^{-b} b^{j+1} / [(j+1)!\,j!] \qquad C_{n+1} = 0 \qquad (27)$$

The error in $F(a, b)$, as given by the simple approximate expression (Eq (24)), increases with the increase of either of the arguments a or b. This error can be reduced below any arbitrary value by raising the degree n of the polynomial in a. The restriction of the domain of $F(a, b)$ within the window w_b helps to keep the degree n at reasonably small values. A number of computational trials were made for different values of b; these trials showed that a suitable value of n which will keep the maximum error in $F(a, b)$ at $a = w_b$, far below one percent is given by:

$$n = \text{integer part of } (b+8) \qquad (28)$$

over the range $0 \leq b \leq 5$ which covers all practical values of the products βX or ϕX.

Simplified calculation of the fluid temperature $T_f(X, t)$

The introduction of the settling-time and memory concepts together with the consequent approximation of the function F are the bases for the simplified calculation method described below:

1. *Zero-input response*: at any time $t < t_{set}$ the zero-input response is given by:

$$T_{f1}(X, t) = T_i \, e^{-\sigma X} [1 - F(\alpha t, \beta X)] \qquad t < t_{set} \qquad (29)$$

The function $F(\alpha t, \beta X)$ is calculated using Eqs (24)–(26) and (28). At times beyond the settling time, T_{f1} will be found from:

$$T_{f1}(X, t) = T_i \, e^{-\sigma X} \qquad t \geq t_{set} \qquad (30)$$

2. *Zero-state response*: the normalized radiant input $R(t)$ may be expressed by a polynomial in time of degree m:

$$R(t) = \begin{cases} \sum_{j=0}^{m} a_j t^j & 0 < t < t_{ss} \\ 0 & t \geq t_{ss} \end{cases} \qquad (31)$$

where the coefficients a_j can be obtained by fitting the curve sketched in Fig 2. Considering typical daily radiation profiles; it was found that m is of the order of 4 to 6. It can be shown, from Eq (31), that:

$$R(t - z) = \sum_{j=0}^{m} A_j z^j \qquad (32)$$

where the coefficients A_j are given by:

$$A_j = (-1)^j \sum_{i=j}^{m} \frac{i!}{(i-j)!\,j!} a_i t^{i-j} \qquad j = 0, 1, \ldots, m \qquad (33)$$

Substituting from Eqs (24) and (32), the integrand of the convolution integral, in Eq (1), will be put in the form:

$$e^{-\gamma z} F(\psi z, \phi X) R(t - z) = e^{-(\gamma + \psi) z} \sum_{i=0}^{m+n} B_i z^i + e^{-\gamma z} \sum_{i=0}^{m} D_i z^i \qquad (34)$$

where:

$$D_i = C A_i \qquad i = 0, 1, \ldots, m \qquad (35)$$

$$B_i = \sum_{j=k_1}^{k_2} \psi^j C_j A_{i-j} \qquad i = 0, 1, \ldots, n+m \qquad (36)$$

and:

$$k_1 = 0, \quad k_2 = i \quad \text{for } 0 \leq i \leq m$$
$$k_1 = i - m, \quad k_2 = \min(i, n) \quad (37)$$
$$\text{for } m < i < (m+n)$$

Consequently, it can be shown, through a lengthy integration and manipulation, that the zero-state response is given by:

$$T_{f2}(X, t) = -e^{-\gamma z} \times \left[e^{-\psi z} \sum_{i=0}^{m+n} G_i z^i + \sum_{i=0}^{m} P_i z^i \right] \Bigg|_{z_1}^{z_2} \quad (38)$$

where:

$$G_i = \sum_{j=i}^{m+n} \frac{B_j j!}{(\gamma+\psi)^{j-i+1} i!} \quad (39)$$

$$P_i = \sum_{j=i}^{m} \frac{D_j j!}{\gamma^{j-i+1} i!} \quad (40)$$

and the two limits z_1 and z_2 are shown, for different cases, in Fig 7.

The following recurrence relations may help to reduce the computational effort:

$G_i = [B_i + (i+1)G_{i+1}]/(\gamma+\psi)$

$i = 0, 1, \ldots, n+m$

$G_{n+m+1} = 0$

$P_i = [D_i + (i+1)P_{i+1}]/\gamma$

$i = 0, 1, \ldots, m$

$P_{m+1} = 0$

It can be seen from Eq (15) that z_1 is equal to zero for all instances during sunlight hours. Thus, Eq (38) reduces to:

$$T_{f2}(X, t) = \gamma(G_0 + P_0) - \gamma e^{-\gamma z_2} \times \left[e^{-\psi z_2} \sum_{i=0}^{m+n} G_i z_2^i + \sum_{i=0}^{m} P_i z_2^i \right] \quad (41)$$

for $t \leq t_{ss}$ and for instances after $t_{ss} + M$, the zero-state response is equal to zero.

3. *Fluid temperature* $T_f(X, t)$: the fluid temperature at any time and any position along the collector is given by:

$$T_f(X, t) = T_{f1}(X, t) + T_{f2}(X, t) \quad (42)$$

where $T_{f1}(X, t)$ is given by Eq (29) or (30), while $T_{f2}(X, t)$ is obtained from Eq (38). The choice between Eqs (29) and (30) and the assignment of the appropriate values for z_1 and z_2 in Eq (38) depend on the time at which $T_f(X, t)$ is to be calculated. A sample of the results is illustrated in Fig 10.

Conclusion

In this paper, two parameters relevant to the dynamic behaviour of the flat-plate solar collector are identified; 'settling time' and 'memory'. The physical meaning of these parameters is brought out by considering the nature of the fluid-temperature response as the resultant superposition of two components; namely the zero-input response, due to the inlet fluid-temperature, and the zero-state response induced by the radiant input. The settling time is the period of time after which the zero-input response attains a limiting constant value given by Eq (17). On the other hand, the memory is a parameter related to the zero-state response. The instantaneous value of the zero-state response, at any instant of time, is shown to be affected only by the radiant input during a definite interval of time just preceding this particular instant. This interval is called the memory since all the earlier radiation history has no contribution to the instantaneous fluid temperature, ie is forgotten. Based on the definitions of the settling time and memory, a simplified computation technique for determining the fluid-temperature response is presented in this paper.

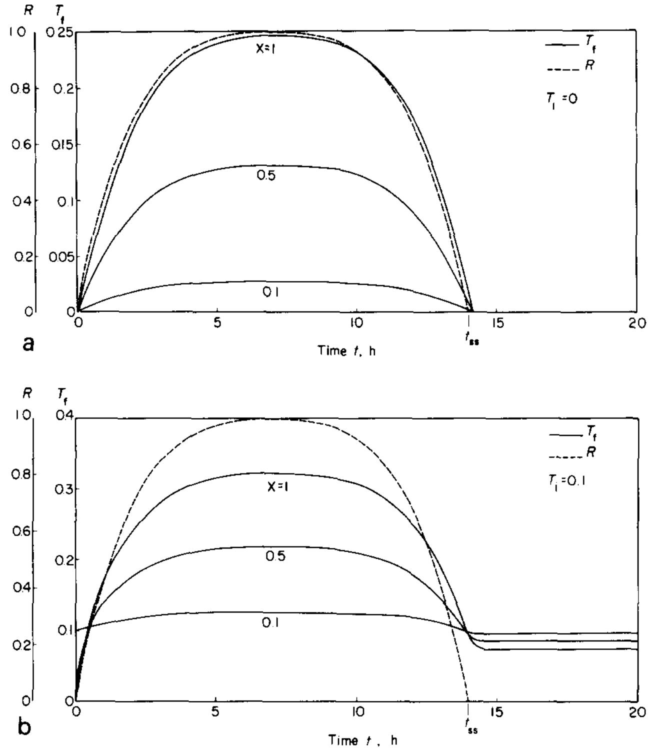

Fig 10 Chronological variation of the fluid temperature at different positions ($\phi = 5$, $\psi = 25$, $\gamma = 1.5$; $R = 0.5091\,t - 0.10019\,t^2 + 0.009118\,t^3 - 0.000326\,t^4$)

A computer program has been written to calculate $T_f(X, t)$ following the proposed simplified computational technique. This program was tested for a number of cases covering the range from extremely short to extremely long settling times and memories. The results were compared with those obtained by other exact methods[1,2]. In all the test cases, with n obtained from Eq (28) and for an accuracy ε of 0.0001, the error in $T_f(X, t)$ was found to be less than 0.1% at all times of the day and all sections of the collector. In practice the ratio H_{max}/U will have a maximum value of the order of 100. Accordingly, and from Eq (11), the error in the actual fluid temperature $\theta_f(X, t)$ will be less than 0.1 °C.

The computation time has been reduced to less than 20% of that required for the numerical integration or infinite summation techniques[1,2]. Most of this saving is achieved in the calculation of the zero-state response T_{f2} since the evaluation of this component represents the major computational effort. In most practical cases, however, errors in $\theta_f(X, t)$ much higher than that mentioned above can be tolerated. Therefore, the value of ε may be taken higher than 0.0001 and the value of n can be reduced below that given by Eq. (28). Consequently, a further saving in the computation time can be achieved.

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